Exam for the M.Sc. in Economics
University of Copenhagen

Political Economics, Fall 2009

Suggested short solutions

1. **Short questions.** Write briefly and concisely, no more than 2 pages per question.

   (a) Define single-peaked preferences and explain why this concept is of interest in political economics modelling.

   Solution: Assume policy $q \in \mathbb{R}$ (a scalar). Voter $i's$ policy preferences are *single peaked* if

$$\left.\begin{array}{l} q'' \leq q' \leq q(\alpha^i) \\ q'' \geq q' \geq q(\alpha^i) \end{array}\right\} \Rightarrow W(q'', \alpha^i) \leq W(q', \alpha^i)$$

   Single-peakedness is one of a few possible requirements to ensure the existence (and uniqueness) of a winning policy (Condorcet winner) under a simple majority rule and to relate this policy to the preferences of median voter (Median Voter Theorem), which is one of key results in (early development) of political economics. To put it differently, it is one of the requirements that ensures existence of a solution when we try to aggregate societal preferences through majority voting procedure. The question is based on PT Ch.2.

   (b) In a median-voter equilibrium of an economy with broad redistributive program higher income inequality leads to more redistribution. True or false? Explain your answer.

   Solution: The effect of higher income inequality on redistribution depends on the form of inequality. More precisely, the median voter's preferred tax rate is

$$\tau^m = \frac{e^m - e}{L_\tau (\tau^m)}.$$

   So if there is a higher income inequality due to *better* relative position of the middle class (poor become extremely poor), it leads to lower taxation/redistribution. If instead there is a higher income inequality due to *worse* relative position of the middle class (rich become extremely rich), it leads to higher taxation/redistribution. This question is based on Ch 6. in PT.

   (c) Suppose two parties i and j are engaged in a war-of-attrition style negotiation over a budget (this is also sometimes referred to as a game of chicken, or a staring contest). If j concedes, i wins and vice versa. If i wins it receives utility U_i^W, if it loses it receives utility U_i^L. Party i does not know the optimal concession time of its opponent, only its associated cumulative distribution function H and the density function h. We can write the expected utility of party i as a function of T_i as

$$EU_i (T_i) = \int_0^{T_i} U_i^W (t) \, h (t) \, dt + (1 - H (T_i)) \, U_i^L (T_i) \, .$$

   Explain why expected utility has this form.

   The marginal cost of continuing bargaining is given by δ_i. If we solve for the optimal stopping time T_i^*, we can show that the (necessary and sufficient) first order condition is

$$\left[u^W - u^L\right] \frac{h (T_i^*)}{1 - H (T_i^*)} = \delta_i.$$

   where the gain from winning $u^W - u^L$ is independent of T_i. Explain the intuition: What is the trade-off faced by party i?

   Solution: The first component of expected utility of conceding at time T_i

$$EU_i (T_i) = \int_0^{T_i} U_i^W (t) \, h (t) \, dt + (1 - H (T_i)) \, U_i^L (T_i) \, .$$

reflects the expected utility of party i in case the other party concedes between 0 and T_i, and, thus party i wins. The first part of the second component, $(1 - H(T_i))$, represents the probability party j does not concede between periods 0 and T_i, and the entire second component reflects the probability for party i to concede first and lose.

This equation states that T_i should be chosen to equate marginal benefit of waiting another instant to concede with the marginal cost of continuing bargaining δ_i. The marginal benefit, in turn, consists of the conditional probability that the opponent will concede "within the next instant", times the gain from winning $[u^W - u^L]$. The question is based on Andersen, Lassen and Nielsen (2009) "Late Budgets".

2. **NOTICE THE TYPO HERE: THE NOTATIONS f_P^{Maj} AND $f_P^{\Pr op}$ SHOULD BE INTERCHANGED IN (a) AND (b), WHICH IS DONE IN THIS SOLUTION.** Consider a society with a politician and N citizens. The politician allocates a fixed budget of size 1. The procedure is as follows: the politician makes a proposal of how much money to give to each individual citizen and to the politician himself, $(f_1, f_2, ..., f_N, f_P)$, such that $f_i \geq 0$ and the budget is balanced

$$f_P + \sum_{i=1}^{N} f_i = 1.$$

The citizens observe the proposal and vote on it, simultaneously and non-cooperatively. If the proposal is rejected, each of the citizen gets a default option of $\overline{f}_i = \frac{1}{N}$, $i = 1, ..., N$, and politician does not get anything. Voting is sincere, that is, each citizen votes for the proposal if and only if it pays as much as the default option.

(a) Assume that in order to get the proposal accepted, the politician needs a support of at least $N/2$ (i.e., a majority) of citizens. Describe all possible Nash equilibria outcomes in terms of allocation of the budget (an informal description is sufficient). What is the budget share of the politician in these equilibria $(f_P^{\Pr op})$?

Solution: In this situation politician needs to "buy" the support of only $N/2$ citizen, the "winning coalition". As the other $N/2$ citizen would be competing to enter the winning coalition, the payment that the members of the winning coalition get from politician will be pressed down to the outside option, $\overline{f}_i = \frac{1}{N}$. The politician, therefore, would get

$$f_P^{\Pr op} = 1 - \frac{N}{2} * \frac{1}{N} = \frac{1}{2}.$$

The set of all equilibria, thus, consists of all allocations in which exactly $N/2$ citizen get $\frac{1}{N}$, and politician gets 1/2. Let's prove it (somewhat) more formally. First, there is clearly no NE in which less than $N/2$ of citizen get transfers of at least $\overline{f}_i = \frac{1}{N}$, as in this case there would be no majority support for the proposal of politician, the default allocation will be realized, and politician will get zero (while she can get 1/2 by proposing allocation as above). Second, there is no NE in which more than $N/2$ citizen get transfers of at least $\frac{1}{N}$. Indeed, the politician can propose an alternative allocation, giving the same transfers to exactly $N/2$ citizen and pocketing the rest of the money, which will yield her support and bring extra payoff. For exactly same reason, politician should not offer more than just enough to get the support, i.e. $\frac{1}{N}$ to the $N/2$ citizen that she chooses to be her support group.

(b) Assume now that citizens are living in 3 districts of equal size $n = N/3$. In order to get the proposal accepted, the politician needs the support of at least 2 (i.e., majority) of the districts. By support of a district here we mean that at least half ($= n/2$) of the district population votes for the proposal. How do the Nash equilibrium allocations of the budget look like? (an informal description is sufficient). What is the budget share of politician f_P^{Maj}?

Solution: Now the politician needs to buy support of half of population in two out of three regions, which makes

$$\frac{n}{2} + \frac{n}{2} = n = \frac{N}{3}$$

citizen's votes to buy. For the same reason as above, all possible NE allocations will look as follows: " For 2 out of 3 regions give $\overline{f}_i = \frac{1}{N}$ to half of region's population, give nothing to the rest of population; the reminder of the sum

$$f_P^{Maj} = 1 - \frac{N}{3} * \frac{1}{N} = \frac{2}{3}$$

goes to the politician." The proof goes along the same logic as the proof in (a).

(c) In the view of your results above and the models that we discussed in class comment on the impact of different electoral rules on politician's rents.

Solution: The results above suggest that majoritarian system is associated with higher rents appropriated by politician. This is due to the fact that under majoritarian elections the politician has to "buy" support of smaller part of population, which intensifies competition between voters and leaves politician with more rents to herself. However, as demonstrated in Ch. 8 of PT book, the situation reverses once one considers the issue of electoral competition - the electoral competition among the candidates is stiffer under the majoritarian. system, which diminishes political rents. This subquestion is based on PT Ch.8 and 9.

3. Consider an economy populated by two groups of individuals. The share of population in group $J = 1, 2$ is given by α_J, $\alpha_1 + \alpha_2 = 1$, and group 1 is a minority

$$\alpha_1 < \alpha_2.$$

Individuals in group $J = 1, 2$ have preferences over economic outcomes, given by

$$w_J = c + 2\beta_J \sqrt{e}.$$

Here c stands for private consumption and $2\beta_J \sqrt{e}$ represents individual's utility from clean environment, where e denotes the measures of environmental protection undertaken by the government, and β_J is the relative taste for environmental protection. We assume that β_J is the same within each group, but different across groups - individuals in group 1 care more about the environment than individuals in group 2

$$\beta_1 > 1 > \beta_2.$$

We assume that the parameters are normalized in such a way, that

$$\alpha_1\beta_1 + \alpha_2\beta_2 = 1.$$

All individuals have the same income $y = 1$, and are taxed by uniform income tax t levied on everyone, so their consumption is equal to their after tax income

$$c = 1 - t.$$

The government uses the tax proceeds to finance the measures of environmental protection, so that the government budget constraint is given by

$$t = e.$$

(a) Derive the level of taxation t^{SO}/environmental protection e^{SO} in utilitarian social optimum.

Solution: In utilitarian social optimum social planner solves

$$\begin{aligned}
&\max_e \alpha_1(c + 2\beta_1\sqrt{e}) + \alpha_2(c + 2\beta_2\sqrt{e}) \\
&s.to\ c = 1 - t \\
&\qquad t = e
\end{aligned}$$

FOC is

$$\begin{aligned}
&\frac{\partial}{\partial t}(\alpha_1(1 - t + 2\beta_1\sqrt{t}) + \alpha_2(1 - t + 2\beta_2\sqrt{t})) \\
= \quad & -(\alpha_1 + \alpha_2) + \frac{1}{\sqrt{t}}(\alpha_1\beta_1 + \alpha_2\beta_2) = 0
\end{aligned}$$

Taking into account normalizations $\alpha_1 + \alpha_2 = 1$ and $\alpha_1\beta_1 + \alpha_2\beta_2 = 1$, FOC yields that in the social optimum

$$t^{SO} = 1 = e^{SO}$$

(b) Compute each individual's preferred tax level. How does it depend on the taste for environmental protection β_J? Provide intuition to your answer.

Solution: An individual in group $J = 1, 2$ solves

$$\begin{aligned} &\max_e \alpha_J (c + 2\beta_J \sqrt{e_J}) \\ &s.to\ c_J = 1 - t_J \\ &\quad t_J = e_J \end{aligned}$$

FOC is

$$\begin{aligned} &\frac{\partial}{\partial e}\left[\alpha_J(1 - t_J + 2\beta_J\sqrt{t_J})\right] \\ = &-\alpha_J + \frac{1}{\sqrt{t_J}}\alpha_J\beta_J = 0 \end{aligned}$$

which is equivalent to

$$t_J = (\beta_J)^2$$

The individual that cares more about environmental protection (higher β) is also willing to set (and pay) higher tax rate to finance the protection.

(c) Suppose that two purely office-seeking (i.e., maximizing its probability of winning) political parties, A and B, compete in elections in this economy. Assume that each party can commit to its electoral promises in case it wins elections. Citizens vote for the party that provides them with most utility. The party that gets most votes wins, and when there is a tie, each of the parties wins with probability 1/2.

i. What is the level of taxation t^*/environmental protection e^* in this equilibrium?

Solution: In this case the parties will compete for the median voter (as the party that gets the support of the median voter will also get the majority of votes and win the elections). As group 1 is a minority, $\alpha_1 < \alpha_2$, median voter belongs to group 2, and thus the level of taxation t^*/environmental protection e^* in this equilibrium is given by group 2's best preferred policy

$$t^* = e^* = (\beta_2)^2$$

ii. Compare them to the socially optimal levels you found in (a) and comment on the source of difference.

Solution:

$$t^* = e^* = (\beta_2)^2 < 1 = t^{SO} = e^{SO}$$

As citizens who are less interested in environmental protection are majority, their interests are pursued by the politicians in this Downsian electoral competition setting. The resulting policy does not in any way take into account the interests of the minority and deviates down from the first best.

(d) Now assume that voters also have ideological preferences towards parties. Voter i in group J votes for party A if

$$w_J(e_A) > w_J(e_B) + \sigma^i_J + \delta$$

where σ^i_J is an individual ideological taste parameter, and δ is a population-wide ideological shock. Assume that for each group $J = 1, 2$ parameter σ^i_J has a uniform group-specific distribution with density ϕ_J and mean 0, and δ is uniformly distributed with density ψ and mean 0. When parties propose their policies (taxes/environmental protection), they know the distributions of σ^i_J and δ but not the exact values.

i. Identify the swing voter in group J and show that the share of votes party A gets in group J is given by

$$\phi_J\left(w_J(e_A) - w_J(e_B) - \delta\right) + \frac{1}{2}.$$

What is the total share of votes for party A in the entire population? (HINT: remember that group sizes are given by α_J)

Solution: The swing voter in group J is a person who is indifferent between supporting party A and party B, that is a voter with $\overline{\sigma}^i_J$ that satisfies

$$\begin{aligned} w_J(e_A) &= w_J(e_B) + \overline{\sigma}^i_J + \delta \Leftrightarrow \\ \overline{\sigma}^i_J &= w_J(e_A) - w_J(e_B) - \delta \end{aligned}$$

Everyone with $\sigma_J^i < \overline{\sigma}_J^i$ supports party A, and with $\sigma_J^i > \overline{\sigma}_J^i$ supports party B. Therefore the share of the votes party A gets in group J is given by

$$\pi_{AJ} = \int_{\sigma_J^i < \overline{\sigma}_J^i} \phi_J d\sigma_J^i.$$

As σ_J^i has a uniform distribution with density ϕ_J and mean 0, it is equivalent to saying that it is distributed uniformly on $\left[-\frac{1}{2\phi_J}, \frac{1}{2\phi_J}\right]$ and the integral above is equal to

$$\begin{aligned}
\pi_{AJ} &= \int_{-\frac{1}{2\phi_J}}^{\overline{\sigma}_J^i} \phi_J d\sigma_J^i = \phi_J \left(\overline{\sigma}_J^i + \frac{1}{2\phi_J}\right) \\
&= \phi_J \left(w_J(e_A) - w_J(e_B) - \delta + \frac{1}{2\phi_J}\right) \\
&= \phi_J \left(w_J(e_A) - w_J(e_B) - \delta\right) + \frac{1}{2}
\end{aligned}$$

The total share of votes for party A is thus

$$\begin{aligned}
\pi_A &= \alpha_1 \pi_{A1} + \alpha_2 \pi_{A2} \\
&= \alpha_1 \left[\phi_1 \left(w_1(e_A) - w_1(e_B) - \delta\right) + \frac{1}{2}\right] + \alpha_2 \left[\phi_2 \left(w_2(e_A) - w_2(e_B) - \delta\right) + \frac{1}{2}\right] \\
&= \sum_{J=1,2} \alpha_J \phi_J \left(w_J(e_A) - w_J(e_B)\right) - \left(\alpha_1 \phi_1 + \alpha_2 \phi_2\right) \delta + \left(\alpha_1 + \alpha_2\right) \frac{1}{2} \\
&= \sum_{J=1,2} \alpha_J \phi_J \left(w_J(e_A) - w_J(e_B)\right) - \left(\alpha_1 \phi_1 + \alpha_2 \phi_2\right) \delta + \frac{1}{2}
\end{aligned}$$

ii. Show that the probability of winning elections for party A is given by

$$p_A\left(e_A, e_B\right) = \frac{1}{2} + \psi \left(\sum_{J=1,2} \alpha_J \phi_J \left(w_J(e_A) - w_J(e_B)\right)\right).$$

Solution: The probability of winning elections for party A depends on the realization of the population-wide ideological shock δ and is given by

$$\begin{aligned}
& \Pr(\pi_A > 1/2) \\
=& \Pr\left(\sum_{J=1,2} \alpha_J \phi_J \left(w_J(e_A) - w_J(e_B)\right) - \left(\alpha_1 \phi_1 + \alpha_2 \phi_2\right) \delta + \frac{1}{2} > \frac{1}{2}\right) \\
=& \Pr\left(\sum_{J=1,2} \alpha_J \phi_J \left(w_J(e_A) - w_J(e_B)\right) - \left(\alpha_1 \phi_1 + \alpha_2 \phi_2\right) \delta > 0\right) \\
=& \Pr\left(\left(\alpha_1 \phi_1 + \alpha_2 \phi_2\right) \delta < \sum_{J=1,2} \alpha_J \phi_J \left(w_J(e_A) - w_J(e_B)\right)\right) \\
=& \Pr\left(\delta < \frac{1}{\alpha_1 \phi_1 + \alpha_2 \phi_2} \sum_{J=1,2} \alpha_J \phi_J \left(w_J(e_A) - w_J(e_B)\right)\right).
\end{aligned}$$

As δ has a uniform distribution with density ψ and mean 0, it is equivalent to saying that it is distributed uniformly on $\left[-\frac{1}{2\psi}, \frac{1}{2\psi}\right]$ and the probability above is equal to

$$
\begin{aligned}
& \Pr\left(\delta < \frac{1}{\alpha_1\phi_1 + \alpha_2\phi_2} \sum_{J=1,2} \alpha_J\phi_J \left(w_J(e_A) - w_J(e_B)\right)\right) \\
= & \int_{-\frac{1}{2\psi}}^{\frac{1}{\alpha_1\phi_1+\alpha_2\phi_2}\sum_{J=1,2}\alpha_J\phi_J(w_J(e_A)-w_J(e_B))} \psi d\delta \\
= & \psi\left(\frac{1}{\alpha_1\phi_1 + \alpha_2\phi_2} \sum_{J=1,2} \alpha_J\phi_J \left(w_J(e_A) - w_J(e_B)\right) + \frac{1}{2\psi}\right) \\
= & \frac{\psi}{\alpha_1\phi_1 + \alpha_2\phi_2} \sum_{J=1,2} \alpha_J\phi_J \left(w_J(e_A) - w_J(e_B)\right) + \frac{1}{2}
\end{aligned}
$$

iii. Show that taxation t^{**}/environmental protection e^{**} in this equilibrium is given by

$$t^{**} = e^{**} = \left(\frac{\alpha_1\phi_1\beta_1 + \alpha_2\phi_2\beta_2}{\alpha_1\phi_1 + \alpha_2\phi_2}\right)^2 .$$

Solution: FOC is

$$
\begin{aligned}
\sum_{J=1,2} \alpha_J\phi_J \left(\frac{\partial w_J(t_A)}{\partial t_A}\right) &= 0 \Leftrightarrow \\
\sum_{J=1,2} \alpha_J\phi_J \left(\frac{\partial\left[1 - t_A + 2\beta_J\sqrt{t_A}\right]}{\partial t_A}\right) &= 0 \Leftrightarrow \\
\sum_{J=1,2} \alpha_J\phi_J \left(-1 + \frac{\beta_J}{\sqrt{t_A}}\right) &= 0 \Leftrightarrow \\
-\left(\alpha_1\phi_1 + \alpha_2\phi_2\right) + \frac{\alpha_1\phi_1\beta_1 + \alpha_2\phi_2\beta_2}{\sqrt{t_A}} &= 0 \Leftrightarrow \\
t_A &= \left(\frac{\alpha_1\phi_1\beta_1 + \alpha_2\phi_2\beta_2}{\alpha_1\phi_1 + \alpha_2\phi_2}\right)^2
\end{aligned}
$$

iv. Under what condition there is an equilibrium in which there is too much environmental protection from the socially optimal point of view? Provide an intuitive explanation for your result.
Solution: Too much environmental protection means that

$$
\begin{gathered}
t^{**} > t^{so} = 1 \Leftrightarrow \\
\left(\frac{\alpha_1\phi_1\beta_1 + \alpha_2\phi_2\beta_2}{\alpha_1\phi_1 + \alpha_2\phi_2}\right)^2 > 1 \Leftrightarrow \\
\alpha_1\phi_1\beta_1 + \alpha_2\phi_2\beta_2 > \alpha_1\phi_1 + \alpha_2\phi_2 \Leftrightarrow \\
\phi_1\left(\alpha_1\beta_1 - \alpha_1\right) + \phi_2\left(\alpha_2\beta_2 - \alpha_2\right) > 0 \Leftrightarrow \\
\text{because of normalizations } \alpha_1 + \alpha_2 = 1 \text{ and } \alpha_1\beta_1 + \alpha_2\beta_2 = 1 \\
\phi_1\left(\left[1 - \alpha_2\beta_2\right] - \left[1 - \alpha_2\right]\right) + \phi_2\left(\alpha_2\beta_2 - \alpha_2\right) > 0 \Leftrightarrow \\
\phi_1\left(\alpha_2 - \alpha_2\beta_2\right) + \phi_2\left(\alpha_2\beta_2 - \alpha_2\right) > 0 \Leftrightarrow \\
\left(\phi_1 - \phi_2\right) \underbrace{\alpha_2}_{>0} \underbrace{\left(1 - \beta_2\right)}_{>0} > 0 \Leftrightarrow \\
\phi_1 > \phi_2
\end{gathered}
$$

So, given our normalization, a necessary and sufficient condition for too much environmental protection in equilibrium is $\phi_1 > \phi_2$, i.e. more swing voters in the environmentally interested group 1, than in environmentally non-interested group 2.